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##### Existence and multiplicity of solutions

##### for a singular -Laplacian problem

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Abstract. In this article, Using the variational method and critical point theory, we prove

the existence two weak solutions for a -Laplacian boundary value problem with

singular nonlinearities in a smooth bounded domain in .

## 1. Introduction

The quasilinear operator -Laplacian has been used to model steady-state solutions such as reaction-diffusion problems. The differential operator is known as the -Laplacian operator, if .

We point out that in [1] the existence weak solutions for the quasilinear elliptic problem of sigaular -Laplacian is studied and in [3, 4] the authors proved the existence of nontrivial weak solutions of elliptic equations.

Here, we consider the singular -Laplacian elliptic problem as

(1.1)

where , is a real parameter and is a Caratheodory function such that

,

where and are positive constants, . Note that is the critical Sobolev exponent.

Let endowed with the norm

(1.2)

and the norm in is

(1.3)

Assume , the compact embedding shows that there exists a such that

(1.4)

where is the best constant of the embedding.

We recall the classical Hardy’s inequality:

(1.5)

where and , see [5].

If we set , for every , then the energy functional associated with (1.1) can be written

where

such that

By (1.5),

(1.6)

for every .

**Definition 1.1**  *The function is a weak solution of (1.1), if and*

for every .

Since is bounded and , we have and the continuous embedding .

**Definition 1.2**  *A Gâteaux differentiable function satisfies the Palais-Smale condition (in short (PS)-condition) if any sequence such that*

is bounded,

,

has a convergent subsequence.

We need the following proposition and theorem to prove the main result.

**Proposition 1.3**  *The operator defined by*

for every , is strictly monotone.

**Theorem 1.4** *[2, Theorem 3.2] Let be a real Banach space and let be two continuously Gâteaux differentiable functionals such that is bounded from below and . Fix such that and assume that, for each , the functional satisfies (PS)-condition and it is unbounded from below. Then, for each the functional admits two distinct critical points.*

## 2 Main results

The statement of main result is as follows:

**Theorem 2.1**  *Let be a Carathodory function such that condition holds. Moreover, assume that*

There exist and such that

for each and .

Then for each , problem (1.1) admits at least two distinct weak solutions, where

and is the constant of the embedding for each in (1.4).

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   Key words and phrases. -Laplacian operator, Singular problem, Variational methods.

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