The Extended Abstracts of
The 6^{th} Seminar on Functional Analysis and its Applications
4-5th March 2020, University of Isfahan, Iran

ABOUT SUBSPACE-DISKCYCLIC WEIGHTED SHIFTS

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ABSTRACT. In this paper, we investigate subspace-diskcyclicity of weighted shifts. We show that any power of a hypercyclic weighted shift and the direct sum of a hypercyclic weighted shift with itself, are subspace-diskcyclic. Also, we give some sufficient conditions for forward shifts and backward shifts to be subspace-diskcyclic.

1. INTRODUCTION

Let X be a complex Banach space and let $B(X)$ be the set of linear continuous operators from X to X. If $T \in B(X)$, then the orbit of x under T is denoted by $orb(T, x)$ and defined as:

$$orb(T, x) = \{x, Tx, T^2x, ...\}.$$

Mathematicians have various operators according to the characteristics of orbits. For example, we say an operator T is hypercyclic if there exists some vector x such that $orb(T, x)$ be dense in X. Zeana in her Ph.D. thesis [5], defined diskcyclic operators as follows.

Definition 1.1. An operator $T \in B(X)$ is called diskcyclic if there is a vector $x \in X$ such that the disk orbit

$$\mathbb{D}orb(T, x) = \{\lambda T^n(x); \lambda \in \mathbb{C}, |\lambda| \leq 1, n \in \mathbb{N}_0\},$$

2010 *Mathematics Subject Classification.* Primary: 47A16; Secondary: 47B37.
Key words and phrases. hypercyclic operators; weighted shifts ; subspace-diskcyclic operators; diskcyclic operators.

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is dense in X. Such a vector x is called diskcyclic for T, where $\mathbb{D} = \{x \in \mathbb{C} : |x| \leq 1\}$ is the closed unit disk.

It is clear by the definition that hypercyclic operators are diskcyclic. Bamerni, Kilicman and Noorani in [1] proved that if x is a diskcyclic vector for T, then for any $n \in \mathbb{N}$, $T^n x$ is a diskcyclic vector for T. Moreover, one can find some sufficient condition for diskcyclicity in [1].

Weighted shifts are an interesting matter for mathematicians in operator theory. We say thee operator T is a forward weighted shift with respect to the canonical basis $\{e_n : n \in \mathbb{Z}\}$ if $T(e_n) = w_n e_{n+1}$, where the weight sequence $\{w_n : n \in \mathbb{Z}\}$ is a bounded subset of $\mathbb{C} \setminus \{0\}$. Similarly, we define a backward weighted shift T with $T(e_n) = w_n e_{n-1}$.

It is proved in [1] that the hypercyclicity and diskcyclicity of a multiple of a unilateral backward shift on $l^2(\mathbb{N})$ are equivalent.

The concept of subspace-diskcyclicity is defined in [2] as follows.

Definition 1.2. Let $T \in B(X)$ and let M be a closed subspace of X. Then T is called a subspace-diskcyclic operator for M (or M-diskcyclic) if there exists a vector $x \in X$ such that $\mathbb{D}orb(T, x) \cap M$ is dense in M. Such a vector x is called a subspace-diskcyclic (or M-diskcyclic) vector for T.

Authors showed in [2] that subspace-diskcyclic operators exist on every finite-dimensional Banach spaces.

In this paper, we investigate subspace-diskcyclicity of weighted shifts. We show that any power of a hypercyclic weighted shift and the direct sum of a hypercyclic weighted shift with itself, are subspace-diskcyclic. Also, we give some sufficient conditions for forward shifts and backward shifts to be subspace-diskcyclic.

2. Main results

We begin with the following theorem.

Theorem 2.1. *([3]) If $T \in B(X)$ is a diskcyclic operator, then there is a non-trivial closed subspace M of X such that T is M-diskcyclic.*

Now we state our first corollary.

Corollary 2.2. *Let $T \in B(X)$ be a weighted shift. If T is hypercyclic, then T is subspace-diskcyclic with respect to a non-trivial closed subspace M of X.*

Proof. It is said that hypercyclic operators are diskcyclic. Theorem2.1, asserts that diskcyclic operators are subspace-diskcyclic. Especially this is true for diskcyclic weighted shifts. $\square$

Moreover, hypercyclicity of a weighted shift T, conclude subspace-diskcyclicity of $T \oplus T \oplus ... \oplus T$ and T^n as follows.

Theorem 2.3. *Let T be a hypercyclic weighted shift. Then:*

(i) $T \oplus T \oplus ... \oplus T$ *is subspace-diskcyclic.*
(ii) T^n *is subspace-diskcyclic for any* $n \in \mathbb{N}$.

Proof. For proving item (i) note that since T is a hypercyclic weighted shift, we can conclude that $T \oplus T \oplus ... \oplus T$ is hypercyclic by Corollary2.10 of [4]. Now Corollary2.2 completes the proof.

For proving item (ii), by Corollary2.12 of [4] T^n is hypercyclic and by Corollary2.2, it is subspace-diskcyclic. □

Theorem 2.4. *Let $T \in B(X)$ be a forward weighted shift with positive weight sequence $\{w_n\}$. If for given $\varepsilon > 0$ and $q \in \mathbb{N}$, there is n arbitrarily large such that for all $|j| \leq q$ we have*

$$\prod_{s=0}^{n-1} w_{s+j} < \varepsilon \quad \textit{and} \quad \prod_{s=1}^{n} w_{j-s} > \frac{1}{\varepsilon},$$

then T is subspace-diskcyclic with respect to a non-trivial and closed subspace M of X.

Proof. By Theorem2.1 of [4] T is hypercyclic and by Corollary2.2 is subspace-diskcyclic. □

Now we say a corollary from [1].

Corollary 2.5. *If T is a backward weighted shift on $l^2(\mathbb{N})$ with weight sequence $\{w_n\}$, then T is diskcyclic if and only if $limsup_{n\to\infty}(w_1 w_2 ... w_n) = \infty$.*

By Corollary2.5, we can state a sufficient condition for subspace-diskcyclicity of backward weighted shifts.

Corollary 2.6. *Let T be a backward weighted shift on $l^2(\mathbb{N})$ with weight sequence $\{w_n\}$. If $limsup_{n\to\infty}(w_1 w_2 ... w_n) = \infty$, then T is subspace-diskcyclic with respect to a non-trivial and closed subspace M of X.*

In the next theorem, we state a sufficient condition for subspace-diskcyclicity of invertible forward weighted shifts.

Theorem 2.7. *Let $T : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ be an invertible forward weighted shift with weight sequence $\{w_n\}_{n\in\mathbb{Z}}$. If there is a sequence$\{n_m\}$ such that:*

(i) $lim_{m\to\infty} \prod_{k=1}^{n_m} \frac{1}{w_{-k}} = 0$,

(ii) $lim_{m\to\infty}(\prod_{k=1}^{n_m} w_k)(\prod_{k=1}^{n_m} \frac{1}{w_{-k}}) = 0.$
Then T is subspace-diskcyclic.

Proof. By Proposition2.14 of [1] T is diskcyclic and by Theorem2.1 is subspace-diskcyclic. □

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