The Extended Abstracts of
The 6^{th} Seminar on Functional Analysis and its Applications
4-5th March 2020, University of Isfahan, Iran

RECURRENT WEIGHTED SHIFTS AND SUBSPACE-HYPERCYCLICITY

MANSOOREH MOOSAPOOR *, [1]

[1] *Assistant professor, Department of Mathematics, Farhangian University, Tehran, Iran.*
mosapor110@gmail.com; m.mosapour@cfu.ac.ir.

Abstract. In this paper, we consider the subspace-hypercyclicity of recurrent weighted shifts. We show that there are subspace-hypercyclic weighted shifts that are not recurrent. Moreover, we give a sufficient condition for an invertible and recurrent weighted shift operator such that both the operator and its inverse be subspace-hypercyclic.

1. Introduction

Let X be a Banach space. Let T be a bounded linear operator or briefly an operator on X. We say that T is hypercyclic if there exists $x \in X$ such that $orb(T, x)$ is dense in X, where

$$orb(T, x) = \{x, Tx, T^2x, ...\}.$$

On a complete and separable metric space X, this is equivalent to saying that there exists $n \geq 0$ such that $T^{-n}U \cap V \neq \phi$ for any open and nonempty sets U and V.

Definition 1.1. ([3]) We say that an operator T is recurrent if for any nonempty open set U of X, there exists $n \geq 0$ such that $T^{-n}U \cap U \neq \phi$.

2010 *Mathematics Subject Classification.* Primary: 47A16; Secondary: 47B37.
Key words and phrases. recurrent operators; weighted shifts, hypercyclic operators; subspace-hypercyclic operators.

* Speaker.

By Definition1.1, it is clear that hypercyclic operators are recurrent. We say an operator T is subspace-hypercyclic with respect to a closed and nontrivial subspace M, if there exists $x \in X$ such that $\overline{orb(T,x) \cap M} = M$. This notion is introduced by Madore and Mattinez-Avendano in [3]. Bamerni, Kadets and Kilicman showed in [1] that hypercyclic operators are subspace-hypercyclic.

Theorem 1.2. *Every hypercyclic operator T on X is subspace-hypercyclic for a subspace M of X.*

Let $\{w_n\}_{n\in\mathbb{N}}$ be a bounded sequence of positive numbers. We say $T : l^p \to l^p$, $1 \leq p \leq \infty$, is a unilateral backward weighted shift with weight sequence $\{w_n\}$, if for any $n \geq 1$, we have $T(e_n) = w_n e_{n-1}$ and $T(e_1) = 0$, where $\{e_n\}_{n\in\mathbb{N}}$ is the canonical basis. Similarly, we say T, , is a bilateral backward weighted shift with weight sequence $\{w_n\}_{n\in\mathbb{Z}}$, if for any $n \in \mathbb{Z}$, we have $T(e_n) = w_n e_{n-1}$, where $\{e_n\}_{n\in\mathbb{Z}}$ is the canonical basis and $\{w_n\}_{n\in\mathbb{Z}}$ be a bounded sequence of positive numbers.

For any $1 \leq p < \infty$, recurrent weighted shifts exist on every l^p. But there is no recurrent weighted shift on $l^\infty(\mathbb{N})$ or $l^\infty(\mathbb{Z})$([5]).

In this paper, we consider the subspace-hypercyclicity of recurrent weighted shifts. We show that there are subspace-hypercyclic weighted shifts that are not recurrent. Moreover, we give a sufficient condition for an invertible and recurrent weighted shift operator such that both the operator and its inverse be subspace-hypercyclic.

2. Main results

We start with a theorem about unilateral weighted backward shifts.

Theorem 2.1. *([4]) Let T be a unilateral weighted backward shift and I be the identity operator. Then $I + T$ is hypercyclic.*

In the next theorem, we make nonrecurrent operators by weighted backward shifts.

Theorem 2.2. *([5]) Let $T : l^\infty(\mathbb{N}) \to l^\infty(\mathbb{N})$ be a unilateral weighted backward shift and I be the identity operator. Then $I + T$ is not recurrent.*

Now we state our first corollary.

Corollary 2.3. *There are subspace-hypercyclic weighted shifts that are not recurrent.*

Proof. Let T be a unilateral weighted backward shift on $l^\infty(\mathbb{N})$. By Theorem2.1, $I + T$ is hypercyclic. So by Theorem1.2, there is a closed and nontrivial subspace M of $l^\infty(\mathbb{N})$ such that $I+T$ is subspace-hypercyclic with respect to it. But by Theorem2.2, $I+T$ is not recurrent and this completes the proof. $\square$

Costakis and Parissis proved in [2] that for a bilateral weighted backward shift on $l^2(\mathbb{Z}$, hypercyclicity is equivalent to recurrency.

Theorem 2.4. *([2]) Let $T : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ be a bilateral weighted backward shift. Then T is hypercyclic if and only if T is recurrent.*

By Theorem2.4 and Theorem1.2, it is not hard to prove the next corollary.

Corollary 2.5. *Let T be a recurrent bilateral weighted backward shift on $l^2(\mathbb{Z})$. Then T is subspace-hypercyclic.*

In the next theorem, we show that we can find recurrent weighted shift operators such that both the operator and its adjoint are subspace-hypercyclic.

Theorem 2.6. *There are recurrent weighted shift operators such that both the operator and its adjoint are subspace-hypercyclic.*

Proof. Salas showed in [4, Corollary2.3] that there exist weighted shift operators like T such that T and T^* are hypercyclic and hence subspace-hypercyclic. Moreover, hypercyclicity of T and T^* leads to that there be recurrent. $\square$

Also, we have the following theorem about invertible recurrent operators.

Theorem 2.7. *([5]) Let T be an operator. Then T is recurrent if and only if T^{-1} is recurrent.*

Theorem2.7 leads to the next corollary.

Corollary 2.8. *Let T be an invertible recurrent bilateral weighted backward shift on $l^2(\mathbb{Z})$. Then T and T^{-1} are subspace-hypercyclic.*

Proof. By Theorem2.7, T and T^{-1} are recurrent. Now Corollary2.5, completes the proof. $\square$

References

1. N. Bamerni, V. Kadets, A. Kilicman, *Hypercyclic operators are subspace-hypercyclic*, J. Math. Anal. Appl., (435)(2), (2016) 1812-1815.
2. G. Costakis, I. Parissis, *Szemeredi's theorem, frequent hypercyclicity and multiple recurrence*, Math. Scand., (110), (2012) 251-272.
3. B. F. Madore, R. A. Martinez-Avendano, *Subspace hypercyclicity*, J. Math. Anal. Appl., (373)(2), (2011) 502-511.

4. H. N. Salas, *Hypercyclic weighted shifts*, Trans. Amer. Math. Soc., (347)(3), (1995) 993-1004.
5. G. Costakis, A. Manoussos, I. Parissis, *Recurrent linear operators*, Complex. Anal. Oper. Th., (8)(8), (2014) 1601-1643.