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MODES OF USING REGIME SWITCHINGF MODEL ON AMERICAN PUT OPTIONS

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Abstract. In this paper, we study American option problem under different methods and conditions. First, we consider American put option pricing under regime switching model (based on fron-fixing transformation and the calculation of optimal stopping boundary) and by calculating the optimal stopping boundary, we obtain a stable solution. In fact, this solution is the best price in the shortest possible time and has the better consistency, in compare with other methods. Then, by inserting rational parameter under regime switching model and employing a wieghted finite difference method, the problem would be discretized and we check the stability and positivity condition of American option problem, again. By having rational parameters and Thomas algorithm, we simplify the calculations and show that numerical analysis is effective in the stability and consistency of the solution.
Keywords: Regime switching model, Front-fixing transformation, Black-Scholes model, American put option, Finite difference, Optimal stoping boundary, Option pricing.

1. Introduction

In the financial modeling field, the Black-Scholes model plays an important role in determining the price of high-risk assets. The utility of this famous model has been proven as a theoretical basis in financial markets. But this model is unable to predict the important properties

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of incomes from assets and fluctuations of the market. For this reason, many arguments for developing this model, have been done. In the Black Scholes model, the underhying asset is based on the geometric Brownian motion process, in which fluctuations and movements of asset prices are constant. Thus, it can not predict stochastic and dynamic behavior in changing price. This drawback has been overcome with stochastic volatility, jump diffusion and regime switching models. Furthermore, regime switching models are computationally inexpensive compared to stochastic volatility jump diffusion models and have versatile applications in other fields. In this paper we check American put option pricing under Regime Switching model, with two methods. First, we use Multivariable fixed domain transformation and then rationality parameter.

Definition 1.1. An option which can be exercised at any time between the purchase date and the expiration date. Most options in the U.S. are of this type. This is the opposite of a European-style option, which can only be exercised on the date of expiration. Since an American-style option provides an investor with a greater degree of flexibility than a European style option, the premium for an American style option is at least equal to or higher than the premium for a European-style option which otherwise has all the same features. Also called American option.

Definition 1.2. A model used to calculate the value of an option, by considering the stock price, strike price and expiration date, risk-free return and the standard deviation of the stock's return. Also called Black-Scholes model.

$$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{1.1}$$

2. Regime switching model with Multivariable fixed domain transformation

For solving American put option pricing under regime switching model and obtain a numerical solution in the best condition, we consider Front-Fixing transformation on free boundary problem. First we obtain the solution by using discretization and numerical schemes. Next by numerical analysis, check Stability, truncation error and consistency. We show the efficiency and accuracy of the proposed method. The results are compared with other known approaches to show its competitiveness. [1]

$$\frac{u_{i,j}^{n+1}-u_{i,j}^{n}}{k} = \frac{\delta_i{}^2}{2}\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{h^2}$$
$$+ (r_i - \frac{\delta_i{}^2}{2} + \frac{X_i^{n+1}-X_i^{n}}{kX_i^{n}})\frac{u_{i,j+1}^{n}-u_{i,j-1}^{n}}{2h}$$
$$- r_i u_{i,j}^{n} + \sum_{l\neq i} q_{i,l}(\tilde{u}_{l_i,j}^{n} - u_{i,j}^{n}), \tag{2.1}$$

where

$$\tilde{u}_{l_i,j}^{n} \approx p_{l,i}(x_j,\tau^n) = p_l(x_j + ln(\frac{X_i^n}{X_l^n}),\tau^n). \tag{2.2}$$

3. Regime switching model with rationality parameter

we consider American put option problem with rationality parameter under Regime switching model. Discretize the problem by using some theorem and weighted finite difference scheme. To simplify and linearize the problem, it can be solved by Thomas algorithm. Finally, examine the quantitative properties of the weighted design, and show that the use of an appropriate method, as well as numerical analysis, has a direct impact on the stability and compatibility of the solution. [2]

$$f((E-S)^+ - v_i)((E-S)^+ - v_i).$$

After incorporating above term to the system of $PDEs$ satisfied by the call option price in the regime switching model, $i = 1, ..., I$ we obtain

$$\frac{\partial v_i}{\partial \tau} = \frac{\delta^2{}_i}{2}S^2\frac{\partial^2 vi}{\partial S^2} + r_i S\frac{\partial v_i}{\partial S} - r_i v_i + f((E-S)^+ - v_i)((E-S)^+ - v_i)$$

$$+ \sum_{l\neq i} q_{i,l}(v_l - v_i),\ \ S > 0,\ \ 0 < \tau \leq T, \tag{3.1}$$

for i = 1, . . . , I, jointly with the initial and boundary conditions:

$$v_i(S,0) = max(E-S,0), \tag{3.2}$$

$$\lim_{S\to\infty} v_i(S,\tau) = 0, \tag{3.3}$$

$$\frac{\partial v_i}{\partial \tau}(0,\tau) = -r_i v_i(0,\tau) + f(E - v_i(0,\tau))(E - v_i(0,\tau))$$

$$+ \sum_{l \neq i} q_{i,l}(v_l(0,\tau) - v_i(0,\tau)), \ i = 1, ..., I. \qquad (3.4)$$

Conclusion

A new efficient numerical method for solving a class of complex PDE systems with a free boundary arising in the American option pricing problem under regime-switching models is developed. The method is based on multivariable front-fixing transformation. This approach allows to calculate the optimal exercise boundary as a part of the solution that to our knowledge is the first time that occurs for regime switching. The explicit finite difference scheme that is quick and accurate is used for the numerical solution. In part 3 The main result is to combine simultaneously two recent models of American option pricing. On one hand, the regime switching approach fits better the reality of the market. On the other hand, the rationality parameter approach incorporates the emotional behaviour of the trader and allows the computation of the American option price by solving a nonlinear PDE problem. This paper is a summary of the thesis[3]. It shows that the Regime switching model is one of the best models that can be used by different schemes and give the best result.

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